DIAKOPTICS USING FINITE ELEMENT ANALYSIS.

K.Guillouard*, M.F. Wong*, Member IEEE, V. Fouad Hanna*, Fellow Member IEEE,
J. Citerne**

*France Telecom - CNET
**INSA, CNRS, URA 834

ABSTRACT

A diakoptics technique based on the use of finite element method is developed in this paper. This method enables the analysis of large microwave structures by modular computation of subdomains. The proposed algorithm is validated on a test case of a step in width finline discontinuity.

INTRODUCTION

One of the main limits of electromagnetic simulations, that are based on a numerical solution of Maxwell's equations, is that the size of solved systems depends on the size and the complexity of investigated devices. To overcome this problem, a large structure may be divided into several parts that are studied independently from each other and combined to obtain overall performance of the initial structure.

Since few years, some efforts have been devoted to the use of the diakoptics technique in the electromagnetic analysis [1] [2], [3] and [4]. They propose a diakoptics technique in either Transmission Line Matrix or Finite Difference Time Domain simulations but, to our knowledge, no proposal has been given yet for a Diakoptics technique for the Finite Element Method.

We use here a Finite Element Method (FEM) based on the use of the Whitney 1-forms, i.e. edge elements. These elements have been successfully used to characterize any structure geometry without having parasitic solutions [5]. Beyond this already-well-known property, edge elements give further interesting features which will help the introduction of diakoptics concept in FEM. We will first, using edge elements, interpret the FEM discretization from circuit point of view. Then we will show how a structure can be divided into subdomains, analyzed separately and recombined to characterize the whole structure.

THE DIAKOPTICS TECHNIQUE WITH THE FINITE ELEMENT METHOD

Let's recall that edge elements allow a space discretization of the field whose degrees of freedom are the circulation of the field along the edges, in other words, the electro-motive force (e.m.f) or magneto-motive force (m.m.f.). In the E-field formulation, the electric field is described with edge elements and the discretized Maxwell's equations can be written in matrix form as :

$$\begin{bmatrix} Y_{11} & \cdots & Y_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & Y_{ee} & \cdot \\ \cdot & \cdot & \cdot \\ Y_{n1} & \cdots & Y_{nn} \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ \cdots \\ e_e \\ \cdots \\ e_n \end{bmatrix} = \begin{bmatrix} i_1 \\ \cdots \\ i_e \\ \cdots \\ i_n \end{bmatrix} \quad (1)$$

or $[Y].[e]=[i]$

where [e] is the vector of e.m.f (in Volts), [i] is the current vector (in Amperes) and [Y] is an admittance matrix which relates the interaction between edges. In fact, one may interpret with circuit considerations this system and even the way by which it has been assembled from elementary finite element matrices. The edges can be seen as the nodes of an electric network whose multiports are provided by the elements of the mesh. A tetrahedral element can be seen as a 6-port element, each port corresponding to an edge (fig.1).

TU
4D

0-7803-3246-6/96/$5.00 © IEEE

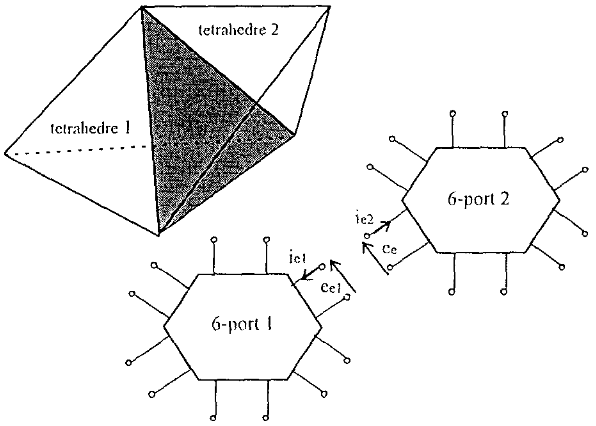


Fig.1 : Comparison of tetrahedral elements with 6-ports.

When connecting the elements together, the common e.m.f vector ensure the continuity of the tangential electric field across common boundary element. The continuity of the tangential magnetic field is verified by writing that the sum of the currents flowing in an edge is equal to a given current i_e. For internal edges, the current is set to zero, while for edges on excitation boundaries, the current is given by a current source (fig.2). It can be seen that the assembly of elementary edge FE is performed in the same manner as is done in circuit nodal analysis.

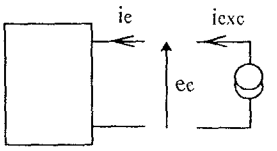


Fig.2 : Edge connected to a current source.

Using this circuit analogy, the diakoptics technique is directly applicable to the Finite Element Method (FEM) based on the edge elements. For the sake of simplicity, let's consider a large structure containing two external ports. It is divided into two subdomains (fig.3).

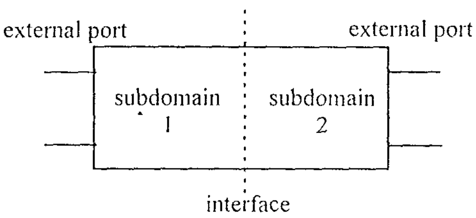


Fig.3 : A 2-port structure.

To simulate directly this structure with the FEM analysis, we have to solve the following system, similar to (1) :

$$\begin{bmatrix} Y_{11} & 0 & Y_{1c} \\ 0 & Y_{22} & Y_{2c} \\ Y_{c1} & Y_{c2} & Y_{cc} \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_2 \\ e_c \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ 0 \end{bmatrix} \qquad (2)$$

where $[e_1]$ and $[e_2]$ are the vectors of e.m.f related to the first and second subdomains respectively except the ones to be introduced at the interface and $[e_c]$ is the vector of e.m.f related to the interface. The same convention is used for the excitation current vectors $[i_1]$ and $[i_2]$.

Now, to simulate separately the two subdomains, we consider the interface as a group of internal ports, each one corresponding to an edge of the interface.

The vector $[e_c]$ becomes $[e_{c1}]=[e_{e1}, e_{e2}, ..., e_{ec/2}]^t$ for the first subdomain and $[e_{c2}]=[e_{ec/2+1}, ..., e_{cc}]^t$ for the second subdomain, supposing that the interface contains c/2 edges (fig.4).

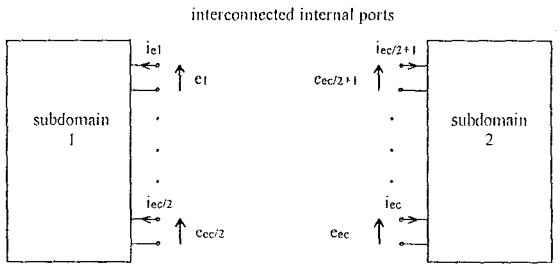


Fig.4 : Interconnection of two subdomains through internal ports.

To express the connection between the two subdomains, we have to satisfy the condition of the electric and magnetic field continuity at the interface. It is equivalent to write the classical Kirchhoff's laws for two interconnected internal ports :

$$\begin{cases} e_c = e_{ci} = e_{ci+c/2} \\ 0 = i_{ci} - i_{ci+c/2} \end{cases} \qquad (3\text{-a})$$

or

$$\begin{cases} [e_c] = [e_{c1}] = [e_{c2}] \\ 0 = [I_{c1}] - [I_{c2}] \end{cases} \qquad (3\text{-b})$$

where $[I_{c1}]=[i_{c1}, i_{c2}, ..., i_{cc/2}]^t$, $[I_{c2}]=[i_{cc/2+1}, ..., i_{cc}]^t$.

The system (2) is divided into the two independent following subsystems characterizing respectively the subdomain 1 and the subdomain 2 :

$$\begin{bmatrix} Y_{11} & Y_{1c} \\ Y_{c1} & Y_{cc1} \end{bmatrix} \begin{bmatrix} e_1 \\ e_{c1} \end{bmatrix} = \begin{bmatrix} i_1 \\ I_{c1} \end{bmatrix} \quad \text{(4-a)}$$

and

$$\begin{bmatrix} Y_{22} & Y_{2c} \\ Y_{c2} & Y_{cc2} \end{bmatrix} \begin{bmatrix} e_2 \\ e_{c2} \end{bmatrix} = \begin{bmatrix} i_2 \\ I_{c2} \end{bmatrix} \quad \text{(4-b)}$$

with $[Y_{cc}]=[Y_{cc1}]+[Y_{cc2}]$.

The two above systems are used separately to perform a FEM simulation of each subdomain, whose impedance matrix is obtained by the classical method described in [5]. It is to be noticed that, in this case, a number of ((2+c)/2) states of independent excitations are considered and that each subdomain contains one external port and c/2 internal ports. For each excitation state, only one external port or internal port is excited by a unit current source while the others are open-circuited. The calculation of the port voltages gives the coefficients of the impedance matrix. We obtain respectively for the subdomain 1 and the subdomain 2 :

$$\begin{bmatrix} \underline{v_{p1}} \\ V_{c1} \end{bmatrix} = \begin{bmatrix} z_{pp1} & \tilde{z}_{pc1} \\ \tilde{z}_{cp1} & Z_{cc1} \end{bmatrix} \cdot \begin{bmatrix} \underline{i_{p1}} \\ I_{c1} \end{bmatrix} \quad \text{(5-a)}$$

and

$$\begin{bmatrix} \underline{v_{p2}} \\ V_{c2} \end{bmatrix} = \begin{bmatrix} z_{pp2} & \tilde{z}_{pc2} \\ \tilde{z}_{cp2} & Z_{cc2} \end{bmatrix} \cdot \begin{bmatrix} \underline{i_{p2}} \\ I_{c2} \end{bmatrix} \quad \text{(5-b)}$$

The voltage vectors $[V_{c1}]$ and $[V_{c2}]$ are directly $[e_{c1}]$ and $[e_{c2}]$. The coefficients v_{p1}, i_{p1} and v_{p2}, i_{p2} are respectively the voltage and current of the external ports. The impedance matrices are composed of three kinds of coefficients : the z coefficients correspond to usual external ports, the Z coefficients correspond to internal ports, the $\tilde{z}$ coefficients are hybrid coefficients.

Using the connection conditions given by equation (3), the independent impedance matrices expressed in (5) are recombined to give the overall impedance matrix of the structure as :

$$[z_p] = \begin{bmatrix} z_{pp1} & 0 \\ 0 & z_{pp2} \end{bmatrix} + \begin{bmatrix} \tilde{z}_{pc1} \\ -\tilde{z}_{pc2} \end{bmatrix} [Z_{cc1} + Z_{cc1}]^{-1} \cdot [-\tilde{z}_{cp1} \quad \tilde{z}_{cp2}]$$

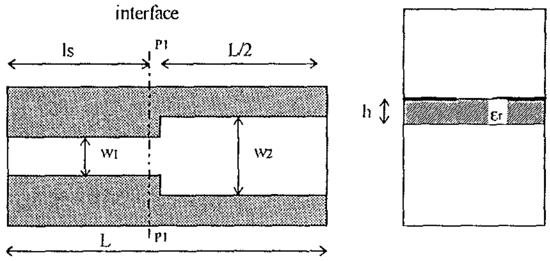
where $\begin{bmatrix} v_{p1} \\ v_{p2} \end{bmatrix} = [z_p] \cdot \begin{bmatrix} i_{p1} \\ i_{p2} \end{bmatrix}$

Only one inversion of a c/2×c/2 matrix is required. The final $[Z_p]$ matrix is entirely equivalent to the one obtained when using directly the system (2) characterizing the complete structure.

The principle of the FEM diakoptics method is shown here on a simple case for the sake of simplicity, but it is adaptable to a structure segmentation into more than two subdomains. This proposed diakoptics technique with FEM analysis shows several advantages. Beside the evident one of allowing larger problems to be solved accurately, another important one is that, if a system is modified, then only the modified portion of the system needs to be solved again. Furthermore, as the overall impedance matrix obtained by our FEM diakoptics technique is strictly equivalent to the one obtained by a direct electromagnetic FEM simulation, the cutting into subdomain may be placed as close as desired to the device to be characterized. This is very useful when characterizing structures having complex internal parts that can be isolated and discretized more precisely. Finally, this technique can be applied on open structures as well as on closed ones.

RESULTS

To validate our diakoptics technique, we study a step in width finline discontinuity like that shown in (Fig.5). We compare the results of two simulations. In the first one (named « direct analysis »), the complete structure is simulated through the usual electromagnetic FEM analysis based on the use of the edge elements. In the second one (named « diakoptics analysis »), the structure is divided into two parts at an interface plane P_1 placed at 0.5mm from the discontinuity. Then, the two parts are simulated separately using the same electromagnetic analysis. The impedance matrix is obtained as explained above and converted to a scattering matrix.



w1=1mm, w2=2mm, L=8mm, ls=3.5mm, h=0.254mm, εr=2.22

Fig.5 : Step in width finline discontinuity.

As expected, the results given in (fig.6) show the strict equivalence of both simulations, even when the interface plane P_1 of the structure is very close to the discontinuity. In the following table, the results of the two simulations are compared to those obtained by a classical modal analysis [6], at a frequency of 29 GHz. The comparison can be judged to be satisfactory.

	direct analysis	diakoptics analysis	modal analysis
$\lvert s_{11}\rvert$	0.2380	0.2380	0.2304
$\lvert s_{12}\rvert$	0.9712	0.9712	0.9785

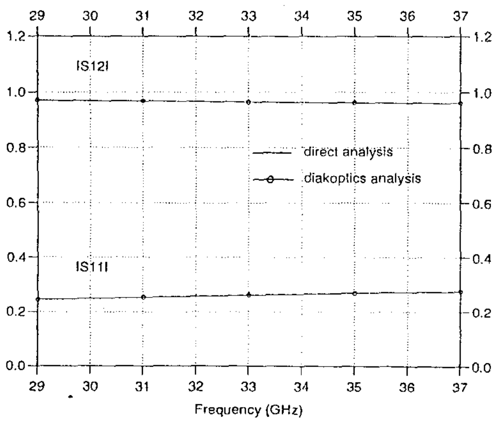


Fig.6 : S-parameters for the structure of (fig.5).

CONCLUSION

The possibility to apply the diakoptics technique, when using a FEM based on the use of edge elements, for characterizing large microwave systems, has been demonstrated. So, the advantages of a FEM analysis, mainly accuracy and possibility of characterizing arbitrary shaped structures, can be maintained, while reducing its drawbacks as modular subdomains are analyzed. The efficiency of the method has been demonstrated on a test case of a step in width finline discontinuity.

REFERENCES

[1] G. Kron : « Diakoptics-The piecewise solution of large scale systems », Macdonald, 1963.

[2] T-W Huang, B. Houshmand, T. Itoh, « The FDTD Diakoptics method », IEEE MTT-S Digest, 1993, pp.1435-1438.

[3] M.Righi, M.Mongiardio, R. Sorrentino, W.J.R. Hoefer, « Efficient TLM Diakoptics for separable structures », IEEE MTT-S Digest, 1993, pp.425-428.

[4] Eswarappa, G.I.Costache, J.R. Hoeffer, « Transmission line matrix modeling of dispersive wide-band absorbing boundaries with time-domain diakoptics for S-parameter extraction », IEEE MTT, April 1990, vol. 38, n°4, pp. 379-386.

[5] M.F. Wong, O. Picon, V. Fouad Hanna, « Three dimensional finite element analysis of N-port waveguide junctions using edge elements », IEEE MTT-S Digest, 1992, pp. 417-420.

[6] M. Hélard, J. Citerne, O. Picon, V. Fouad Hanna, « Theoritical and experimental investigation of finline discontinuities », IEEE MTT, October 1985, vol. 33, n°10, pp.994-1003.